CHAPTER 1

A Precocious Genius

Shadowed by Illness

Blaise Pascal was born on 19 June 1623 in the city of Clermont, the only son of Etienne Pascal and Antoinette Begon. He had an elder sister, Gilberte, born in 1620, and a younger one, Jacqueline, who was two years his junior.[1] His grandfather was a tax official who became private secretary to Queen Louise, the wife of King Henri III, before becoming the Treasurer of France for the Province of Auvergne. Blaise's father, Etienne, was a Presiding Judge at the taxation court in Montferrand, the twin city of Clermont, and, after a period of retirement in Paris, was appointed head of the taxation office in Rouen by Cardinal Richelieu.

Pascal was later to have serious medical problems, which were foreshadowed at an early age. When he was two years old, he became extremely thin and his parents feared for his life. The symptoms were most unusual: he could not look at water without having a fit, and although he was happy when his mother or his father came to him separately, he could not bear seeing them together. In an age prone to see the hidden

[1]Our main source is the *Life of Pascal* written after his death by his sister, Gilberte Périer. It was published for the first time, in a pirated edition, in Amsterdam in 1684, and two years later in Paris, the year before Gilberte died. A second version was discovered and published by Léon Brunschvicg in the twentieth century. Both are included in the standard edition of Pascal's works, the *Œuvres Complètes de Pascal*, edited by Pierre Mesnard. Paris: Desclée de Brouwer, 1964, vol. I, pp. 571–602 and pp. 603–642. Four volumes of this new edition have appeared to date and cover the period 1623–1662, but do not include the *Pensées* or the *Provinciales*. This edition will be quoted as *Œuvres de Pascal* followed by the volume in Roman and the page in Arabic numbers.

hands of witches, it was rumored that a curse had been placed on him by an old woman who belonged to the poor that his family helped.[2] Examined and threatened by the boy's father, the woman broke down and confessed that she had indeed placed a spell on the boy because the judge had refused to arrange some legal matter for her. She declared that the spell was deadly and could only be canceled by some other death, but she promptly added that any animal would do. Pascal's father offered a horse, but she replied that a cat would be more than enough. She was immediately given one, but as she walked out of the room and down the staircase, some servants insulted her and in her excitement she flung the cat out of the window. Although it did not fall from any great height, it landed on its back and was pronounced dead upon striking the ground. This was considered most amazing, because it was well known that a cat that is dropped upside down invariably lands on its four feet.[3] After the old woman recovered her composure, she went out into the garden to gather some herbs, which she mixed with flour to make a poultice.[4] No sooner was it applied to Blaise's stomach than he fell into a coma.

The doctors, summoned to his bedside, declared that the boy was dead, but his father refused to have him made ready for burial. He summoned the old woman, and when she entered the room, he went wild, rushed at her, and slapped her face. She meekly replied that she was only getting what she deserved because she had forgotten to mention that the boy would become unconscious and would only revive around midnight. The family anxiously awaited the outcome, but when the clock struck twelve, Blaise did not stir. Then around two in the morning, he opened his eyes and saw his mother and father together without dread, "which proved that he was cured," comments Gilberte.[5] A few days later his fear of water

[2]The story was told by Gilberte Périer while Pascal was still alive but it was only published in the nineteenth century. Mesnard gives it in *Œuvres de Pascal*, I, pp. 507–508. An enlarged and moralized version is to be found in the *Memoirs* that Marguerite Périer dictated around 1723 when she was already 76 years old and somewhat given to hyperbole (*Œuvres de Pascal*, I, pp. 1091–1093).

[3]I do not recommend carrying out the experiment, but the physical explanation is quite straightforward: by pulling in its paws, the cat is able to rotate very fast for the same reason that a skater can speed up his rate of spinning by bringing his arms in.

[4]Marguerite Périer adds that the witch told Pascal's father that three different kinds of herbs had to be gathered by a child under seven years of age. The local apothecary offered the services of his daughter and brought her early on the next morning (*Œuvres de Pascal*, I, p. 1092).

[5]*Œuvres de Pascal*, I, 508.

subsided and he was soon splashing about in his bath. As we can see, this bizarre incident tells us more about belief in witchcraft in the seventeenth century than about the nature of the aliment that may have been at the origin of Pascal's subsequent illness.[6]

Pascal's mother died in 1626 and shortly thereafter his father resigned as President of the taxation court to concentrate on the education of his three children. The family moved to Paris in 1631, and from that time until 1640, when Pascal's father was sent to Rouen as tax officer by Cardinal Richelieu, Blaise studied at home under his supervision. The father did not want to teach his son subject matters that were beyond his ability: he introduced him to Latin when he was twelve years old, once Blaise had understood that the rules of grammar can be used as keys to unlock the meaning of language. The boy was thus spared seeing grammar as a tedious collection of rules to be memorized and, as his sister Gilberte put it, "when he had to learn it, he knew why." In the colleges run by the Jesuits, boys were taught Latin and Greek at a much earlier age.

Pascal's father was not only a distinguished lawyer but an amateur scientist, and he welcomed every opportunity to interest his son in curious facts of nature. One day, when the family was at table, a piece of china was accidentally struck with a knife and began to hum. When Pascal placed his hand on the plate, he noticed that the noise stopped immediately. This puzzled him and he began experimenting with other forms of sound, and even wrote a paper (which Gilberte graces with the word *treatise*) on his results when he was only eleven.

Euclid Mastered

Pascal's father was acquainted with the leading mathematicians in France and he made important contributions of his own. He studied the curve that

[6]We find the following clinical remark in Gilberte's account: "He was as thin as children who have *chartre*," a condition that is described by Furetière in his *Dictionnaire universel* (1690) as "an illness that leads to passivity and gradual loss of weight, and is also known as *phtisis*," namely tuberculosis (cited in *Œuvres de Pascal*, I, p. 1091, n. 1). Marin Mersenne, who suffered from skin rashes all his life, mentions women in his native village who treated such complaints by blowing on the infected area two or three times in the form of a cross and reciting the Our Father (*Quaestiones in Genesim* (1623), col. 599, cited in French translation in Armand Beaulieu, *Mersenne, le Grand Minime*. Bruxelles: Fondation Nicolas-Claude Fabri de Peiresc, 1995, p. 7).

the Ancients called the *conchoid of the circle* with such success that it was renamed *the snail of Pascal*, as it is still known today. In modern notation it can be written as $r = a + b\ cos\theta$. But Etienne Pascal did not want his son to become engrossed in mathematics before he had mastered Latin and Greek, and he put his books on the subject out of the boy's reach. "What is mathematics and what is it all about"? Blaise persisted in asking his father, who finally replied that it was "a general method of tracing precise figures and finding their ratios," but that he was not to think about it. The result of such provocation for a gifted child may well be imagined. Blaise thought about it furiously. With a piece of charcoal, he drew figures on the floor-tiles, and he tried to draw perfect circles or trace triangles with equal sides. Since he did not even know the names of geometrical properties, he invented his own mathematical terms, calling, for instance, a circle a *round* and a line a *bar*.

One day, when Blaise was absorbed in studying a geometrical figure, his father walked into the room and asked him what he was doing. In his homespun terminology, Blaise explained that he had just found how to show that the sum of the three interior angles of a triangle is equal to two right angles. His father immediately recognized this as a Proposition of Book I of Euclid's *Elements*, and asked what had made him look for such a proof. Blaise answered that it was because he had found such and such a geometrical property, and when his father asked him again how he had discovered this, he mentioned yet another geometrical property until, step by step, he was led back to his initial definitions and axioms.[7] Etienne Pascal was so moved by his son's astonishing performance that he rushed to the house of his close friend, Jacques Le Pailleur, and told him, "with tears in his eyes," what he had just witnessed.

Gilberte does not describe her brother's do-it-yourself method but the Proposition that he reached is number 32 of Book I of the *Elements* and, if he followed the same route as Euclid, he was led to prove that the sum of the three interior angles of a triangle is equal to two right angles after realizing that if any side of a triangle is produced, the exterior angle is equal to the sum of the two interior and opposite angles. This property follows from Proposition 29 of Book I, which uses the postulate of parallel lines to show that a straight line that falls on two parallel straight lines makes the exterior angle equal to the interior and opposite angle on the

[7]*Œuvres de Pascal*, I, p. 606, and pp. 574–575 for the first version.

same side, and hence that the two interior angles on the same side are together equal to two right angles.

It was clear that there was no point in keeping Blaise from mathematics, and he was given a copy of Euclid's *Elements*. Tallemant des Réaux, the author of a collection of anecdotes about famous people of his acquaintance, goes further and claims that Pascal read the first six books of Euclid entirely on his own. This is the kind of embellishment that someone who has no talent for mathematics might wish to ascribe to those who seem to him to penetrate the mysteries of geometry without effort, and it should be taken with a grain a salt. But Blaise was undoubtedly brilliant and he was admitted, probably when he was no more than fourteen or fifteen, into a select discussion group that met in the convent of the friar Marin Mersenne, near the Place Royale (now Place des Vosges) in Paris.

Mersenne had become the clearing-house of the world of science and mathematics, and in addition to bringing scholars and scientists together each week, he wrote to all and sundry, in his cramped handwriting, on what was new and significant. In the absence of learned societies, professional journals, and other means of exchanging ideas, Mersenne made a valuable contribution to the development and dissemination of scientific ideas. He soon became an outspoken admirer of Pascal's mathematical gifts, and he was later to advertise his work on the vacuum. It is at Mersenne's weekly meetings that Pascal met such remarkable personalities as Gilles Personne de Roberval, the professor of mathematics at the Collège Royal; Ismael Boulliau, an eminent astronomer; Pierre Gassendi, the leading French exponent of atomism; Claude Mydorge, a distinguished mathematician with whom Descartes had worked on optics; Pierre Petit, a royal engineer; and Pierre Carcavy, the future librarian of Colbert and Louis XIV.[8] The most important contact for Blaise was Girard Desargues, who had just published a slim volume with the forbidding title, *Brouillon project d'une atteinte aux évènemens des rencontres du Cone avec un Plan*, which can be rendered as *Rough Draft of an Attempt to Deal with the Outcome of the Meeting of a Cone with a Plane*. Why he studiously

[8]Michel de Marolles, who met Pascal at the Academy, states in his *Mémoires* published in 1656 that the meetings were held on Saturday in the apartment of Jacques Le Pailleur. Further evidence is provided by the list of written or intended mathematical works that Pascal submitted to the *Parisian Academy of Mathematical Sciences* in 1654, and in which he says that he was educated in their midst (*Œuvres de Pascal*, II, p. 1031). Le Pailleur died on 4 November 1654. See Jean Mesnard, "Pascal à l'Académie Le Pailleur" in René Taton (ed.), *L'Oeuvre Scientifique de Pascal*, Paris: Presses Universitaires de France, 1964, pp. 7–16.

avoided the obvious title, *Conics*, which Apollonius of Perga had given to his classical work on the subject, is unknown. Desargues laid the foundations of a new kind of geometry, but lost most of his readers in a maze of botanical terms, by calling *trunk* a straight line with points on it, *knots* points on a line through which pass other lines, *branch* each of these lines, and *twig* a segment on one of the lines.[9]

The idea on which Desargues' work is based is a simple observation: a circle, when viewed obliquely, has the shape of an oval. For instance, the shadow of a lampshade looks like a circle on the ceiling, and like an oval where it falls on the wall. This illustrates the fact that shapes and sizes change according to the way light rays fall on them. Throughout these changes, certain properties remain the same, and it is these that Desargues studied. He realized that a conic section stays a conic section no matter how many times it undergoes a projection. In order to use this feature, Desargues took the original step of assuming that a parabola has a locus "at infinity," and that parallel lines meet at "a point at infinity." The theory of perspective, developed in the Renaissance, had made such ideas plausible by considering rays of light from the sun as parallel, and treating them as a cylinder or a parallel pencil of lines, unlike rays from a terrestrial source that they treated as a cone or a point pencil. In Desargues' view, the cylinder is merely a cone whose vertex is infinitely distant, and a parallel pencil of lines is simply a family of lines, all of which go through the same point at infinity.

It is in this way that the young Pascal became interested in geometry, the most concrete of the mathematical disciplines, and specifically in one of its most concrete branches, what we now know as projective geometry. Whereas his older contemporary, Descartes, favored the abstract approach of algebraical analysis, Pascal preferred drawing figures that could be visualized in space, and he enjoyed studying the transformations that result from moving them about. The conic sections were particularly interesting in this respect since the circle, the ellipse, the parabola, and the hyperbola result from different ways of slicing a cone. Following Desargues, Pascal identified parallel and concurrent straight lines because the first can

[9]Desargues did not expect a large readership and had only 50 copies printed in 1639. All were lost, but a transcript made by Philippe de la Hire in 1679 was discovered in Paris in the nineteenth century. In 1951, an original copy was found in the Bibliothèque Nationale in Paris and was published by René Taton in *L'oeuvre mathématique de G. Desargues*, Paris: Presses Universitaires de France, 1951.

be derived from the second when they are projected to infinity. Likewise, he considered a straight line as the limiting case of a circle whose center is located at infinity, and a cylinder the limiting case of a cone whose summit is at an infinite distance. The notion of infinity, which was necessary for this operation, remained incomprehensible, but it ushered in the idea of paradox that we will later find associated with Pascal's notion of scientific proof.

Pascal's First Poster

Pascal was only fifteen when Desargues' book appeared, but in less than a year he published his own original developments in a short but remarkable paper, the *Essay on Conics*, which consisted of a large printed page 35 × 43 centimeters. It is what we would now call a poster,[10] and it contains a proposition, described as the *Mystic Hexagon* that has since been known as Pascal's theorem. In essence, it states that the opposite sides of a hexagon inscribed in a conic section intersect in three collinear points. Pascal expresses this in a somewhat different way by saying that if A, B, C, D, E, and F are successive vertices of a hexagon in a conic, and if P is the intersection point of AB and DE, and Q the point of intersection of BC and EF (see Figure 1 where the conic is the egg-shaped figure, and ABCDEF is a plane figure with six sides),[11] then the lines PQ, CD, and FA "are of the same order" or, as we would put it, are members of a pencil, whether a point pencil or a parallel pencil. Pascal deduced several applications from this proposition, the first being the construction of the tangent to a conic section, but these developments do not appear in the poster.

Marin Mersenne was so impressed that he wrote to a correspondent in London that an eighteen-year-old mathematician (Pascal was actually seventeen at the time) had been able to capture the essential property of conic

[10]The *Essay on Conics* is published in the *Œuvres de Pascal*, II, pp. 228–235, and is translated in D.J. Struik (ed.), *A Source Book in Mathematics 1200–1800*. Princeton University Press, 1986, pp. 163–167. Pascal's poster suffered the same fate as Desargues' book and soon got lost. It was rediscovered by Colin Maclaurin in 1727. See René Taton, "L'oeuvre de Pascal en géométrie projective," *Revue d'histoire des sciences* 15(1962), pp. 197–252, reprinted in *L'Oeuvre Scientifique de Pascal*. Paris: P.U.F., 1964, pp. 17–72; René Taton, "*L'Essay pour les Coniques de Pascal*," *Revue d'Histoire des Sciences* 8(1955), pp. 1–18.

[11]I use the figure as redrawn by Carl B. Boyer in his *History of Mathematics*, Princeton: Princeton University Press, 1985, p. 396.

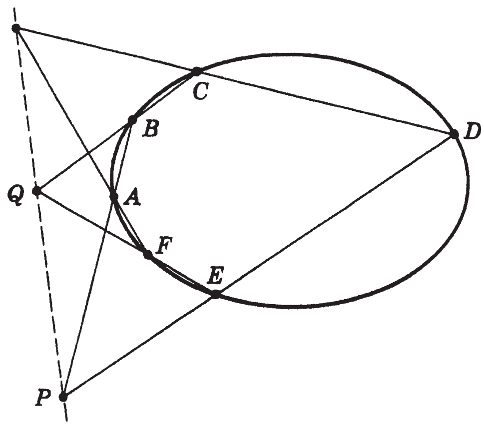

FIGURE 1 *Diagram of Pascal's theorem written when he was seventeen years old.*

sections in a single proposition from which he drew 400 corollaries.[12] He also mentioned the boy-prodigy to Descartes, who replied, rather ungraciously, "I am not surprised that someone can provide demonstrations that are easier than those of Apollonius, a prolix and confused writer who only proved what is quite easy, but there are other things concerning conics that would give a sixteen-year-old child a harder time."[13] As he had promised in his poster, Pascal wrote a treatise on conics, the *Conicorum Opus Completum*, which was not published in his lifetime. The manuscript was loaned to Leibniz by Pascal's heirs when he visited Paris in 1676 and, after taking notes, he made a detailed description of the content, and returned it with the recommendation that it be printed. Unfortunately, Leibniz's advice was never followed, and only the first part, *On Conic Sections*, has survived. It was found among Leibniz's papers after his death and published in 1892 along with Leibniz's notes.

Pascal used his new method to construct tangents to a circle and tangents to a sphere. The first had been studied by Apollonius in Antiquity,

[12]Letter of Mersenne to Theodor Haak, 28 November 1640, *Correspondance du P. Marin Mersenne*, edited by P. Tannery, C. de Waard et al., 16 vols. Paris: Editions du C.N.R.S., 1933–1986, vol. XI, p. 429. The date is erroneously given as 18 November 1640 in the *Œuvres de Pascal*, II, p. 239.

[13]Letter of Descartes to Mersenne, 25 December 1639, *Œuvres de Pascal*, II, p. 237. Apollonius of Perga, who lived in the third century B.C., wrote a treatise on conic sections that is one of the greatest mathematical works from the ancient world.

and was known through the summary that is found in Pappus, a fourth-century mathematician. In 1600, Viète had tried to reconstruct Apollonius' lost procedure in a book that he entitled *The French Apollonius*. Pascal now proposed to generalize the whole method by formulating the problem as follows: Given any three elements (points, lines, or circles), draw a circle that passes through the given points and is tangent to the given lines and circles. The second problem, that of the construction of tangents to a sphere, "is derived," writes Pascal, "from a single property of conic sections that is useful in coping with many other difficult problems. Yet it barely fills a page."[14] Unfortunately, Pascal does not tell us what this property is, and we are left to wonder.

Pascal did not realize that the great developments in mathematics in the seventeenth century were occurring in analytic geometry. François Viète (1540–1603) had seen that the wave of the future was algebraic geometry, and the two leading mathematicians of Pascal's day, Descartes and Fermat, followed his lead. Descartes' *Geometry* was published in 1637, when Pascal was fourteen years old, but the work had less of an impact than might be expected because his co-ordinate system was not clearly explained. Indeed, the phrase, "Cartesian co-ordinates," so often used today, is an anachronism. We do not find in Descartes' *Geometry* the "X-axis" and "Y-axis" with which we are familiar from high school geometry. Descartes' innovation was only fully appreciated after Frans van Schooten published a Latin translation of the *Geometry* with commentaries and additional material in 1649. This may be the reason why Pascal failed to see the extent to which the algorithms of algebra opened a royal road for geometry.

The Calculating Machine

Blaise's father, Etienne, was appointed tax officer in Rouen in 1640 after a rebellion of local peasants had led to the destruction of the registers. The accounting system had to be completely overhauled, and the task took more time and was more onerous than anticipated. Three years later, Pascal's father was still computing well into the night, as we know from a postscript to a letter to his daughter, Gilberte: "My loving daughter will excuse me if I do not write as I should wish to do if I had time. I have never

[14]Pascal, *Letter to the Parisian Academy of Mathematical Sciences*, written in 1654, *Œuvres de Pascal*, II, p. 1033.

faced such difficulties as at present, and I could not cope with more. For four months now, I have not gone to bed more than six times before two o'clock in the morning."[15]

Pascal, who was still in his teens, offered to help his father and was soon confronted with the tedious calculation of fiscal charges. In order to afford relief to his father as well as himself, he began looking into the possibility of mechanical calculation, a task rendered more complicated by the fact that French currency was not computed in tens but by a method like the one that prevailed in the United Kingdom until as late as 1971. The French *livre* consisted of 20 *sols*, and each *sol* was equivalent to 12 *deniers*. The meant that the livre was worth 240 deniers, not 100, and this gave Pascal a headache. After several trials, he invented a system to add and subtract with a series of gears that went forward, but not backward. This was similar to the mechanical calculating machines that served as precursors to today's electronic calculators.

The idea itself is easy to grasp. Pascal placed side by side in an oblong box eight small drums, around the upper and lower halves of which the numbers 0 to 9 were written in descending and ascending orders, respectively (see Figure 2). The upper row of figures was for subtraction, the lower for addition. One half of each drum was shut off from outside view by a sliding metal bar. Below each drum was a wheel consisting of ten (or twenty or twelve) movable spokes inside a fixed rim numbered in ten (or more) equal sections from 0 to 9, rather like a clockface. The numbers to be added or subtracted were fed into the machine by means of wheels on the box lid that were turned with a metal hook. A simple gearing mechanism of five toothed wheels linked the ten movable spokes to their appropriate drum. To increase by one the number showing in any aperture, it was necessary to turn the appropriate drum forward 1/10th of a revolution. For instance, to add 315 + 173, first 315 was recorded on the three drums closest to the right-hand side with 5 appearing in the sighting aperture to the extreme right, 1 next to it, and 3 next to that one. Then 173 was entered by giving the drum on the extreme right three turns, moving the drum to its left by 7/10 of a revolution, and the drum to this one's left by 1/10. The total of 488 could then be read off in the appropriate slots. But, easy as this operation was, a problem clearly arose when the numbers

[15]Postscript to the letter of Blaise to Gilberte, 31 January 1643, *Œuvres de Pascal*, II, p. 283; the letter is translated in Emile Cailliet and John C. Blankenagel (eds.), *Great Works of Pascal*. Philadelphia: Westminster Press, 1948, pp. 39–40.